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APPLICATION OF THE PERTURBATION METHOD TO THE THEORY OF TORSION OF ELASTO-PLASTIC BARS*

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A general approach to solving the problem of finding an elasto-plastic boundary during twisting the elasto-plastic bars is considered. It is assumed that the boundary of transverse cross section of the bar is a smooth curve. The defining relations are derived using the assumption that the angle is small and that the conditions of coupling at the elasto-plastic boundary hold. A method of small parameter is used. The perturbation method has been used before to solve a number of elasto-plastic problems in which the whole contour of the body was surrounded by a plastic zone; they can be found in the monograph /1/. If the plastic flow begins at some point of the contour, then the method requires a certain specified modification, which is given below for the case of torsion of elasto-plastic bars. Approximate solutions were constructed in /2,3/ for elasto-plastic bars of polygonal cross section, using the function of complex variable methods.

1. Consider the torsion of a rectilinear cylindrical bar made of perfect elasto-plastic material, with transverse cross section D and boundary L. We assume that an elastic solution for the bar in question is known. Let $\tau = \{\tau_\alpha\}$ $(\alpha = 1, 2)$ denote the tangential stress appearing in the cylinder during torsion and $\gamma = \{\gamma_\alpha\}$ be the total deformation composed of the elastic γ^e and plastic γ^p component

$$\gamma_\alpha = \gamma_\alpha^e + \gamma_\alpha^p \tag{1.1}$$

In the case of elastic torsion of a bar the deformation tensor components are connected with the displacement by the relation

$$\gamma_\alpha = {}^1/_2\omega\,(\varphi_{,\alpha} + \varepsilon_{\beta\alpha}x_\beta) \tag{1.2}$$

Here ω denotes the twist, $\varphi(x_1, x_2)$ is the St. Venant stress function, $\varepsilon_{\beta\alpha}$ is the antisymmetric unit tensor and x_β is the coordinate of the point at which the deformation is determined. When plastic regions appear in the bar, the total deformation components in these relations will have the form

$$\gamma_\alpha = f_{,\alpha} + \omega\varepsilon_{\beta\alpha}x_\beta \tag{1.3}$$

where $f(x_1, x_2, \omega)$ is the function characterizing the deplanation of the transverse cross section of the bar. The following equation of equilibrium will hold in the elastic and plastic zone of the bar:

$$\tau_{\alpha,\alpha} = 0 \tag{1.4}$$

The stresses and elastic deformations are connected by the Hooke's Law

$$\tau_\alpha = 2\mu\gamma_\alpha^e \tag{1.5}$$

At the bar boundary L the following boundary condition must hold:

$$\tau_\alpha n_\alpha = 0 \tag{1.6}$$

where n_α are components of the unit vector normal to the contour L of the transverse cross section. The stresses appearing in the plastic region of the bar satisfy the condition of plasticity

$$\tau_\alpha\tau_\alpha = k^2 \tag{1.7}$$

and we have the associated flow rule

$$\gamma_\alpha^{\cdot} = \lambda\tau_\alpha \tag{1.8}$$

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where a dot denotes the derivative of the total deformation components with respect to ω. The condition of conjugation of solutions must hold at the boundary L^S separating the elastic and plastic region

$$[\tau_\alpha] = [\gamma_\alpha] = 0 \tag{1.9}$$

2. Conditions (1.4) and (1.7) must hold in the plastic zone, i.e. the problem is statically determinable and its solution has the form /4/

$$\tau_\alpha = ks_\alpha, \quad s_\alpha = \{-\sin\theta, \ \cos\theta\} \tag{2.1}$$

where s_α are the compoents of the unit vector tangent to the contour L, and θ is the angle of inclination of rectilinear stress field characteristics to the x_1-axis. In the present case it coincides with the angle of inclination of the unit normal $n_\beta = \varepsilon_{\beta\alpha}s_\alpha$ of the contour L to the x_1-axis.

Using the relation of the associated law of flow (1.8) and the expression (2.1), we can write the following expression for the warping function of the transverse cross section of the bar in the plastic region:

$$f^{\cdot} = x_\alpha s_\alpha r + C \tag{2.2}$$

where x_α is the coordinate of the point at which $f^{\cdot}(\omega, x_1, x_2)$ is determined, r is the distance along the normal to L, from the point on the boundary L_S to the point in question, and C is a constant with the value specific along each characteristic for a given ω.

Integrating (2.2) we can obtain the warping of the transverse cross section of the bar under torsion $f(\omega, x_1, x_2)$ in the plastic region, provided that the boundary L_S of the elastic region is known. The solution obtained must satisfy the following inequality in the plastic region:

$$\gamma_\alpha^{\cdot}\tau_\alpha \geqslant 0 \tag{2.3}$$

Let the bar under torsion be in the elastic state at $\omega < \omega_0$, and let there exist at $\omega = \omega_0$ at least one point of the bar boundary L at which the plastic state will be realized, i.e. no elastic solution not exceeding the yield point will exist for $\omega > \omega_0$. We shall assume that when $\omega = \omega_0$, a stress will appear at the point A (Fig.1) of the contour, the components of which will satisfy the relation (1.7). Let us choose the Cartesian $x_1 0 x_2$ coordinate system in such a manner, that the x_2-axis passes through this point in the direction perpendicular to the tangent to L at A. We denote by $x_\alpha^{(0)}(\theta_*)$ the coordinates of the point of intersection of the elasto-plastic boundary L_S at $\omega > \omega_0$ with the bar contour. Taking into account the relations (1.2), (1.5), (1.9) and conditions (2.1), we can write the following relation at the boundary L_S:

$$\mu\omega(\varphi_{,\alpha} + \varepsilon_{\beta\alpha}x_\beta) = ks_\alpha \tag{2.4}$$

The equation of the elasto-plastic boundary L_S can be written in the form

$$x_\alpha(\theta) = x_\alpha^{(0)}(\theta) - r(\theta)n_\alpha \tag{2.5}$$

where $x_\alpha^{(0)}(\theta)$ denote the coordinates of the points on the boundary L of the bar transverse cross section and $r(\theta)$ is the magnitude of the segment along the normal to L originating at L. Substituting into (2.4) relation (2.5) and the expansion of φ into a Taylor series along the normal to L, we obtain

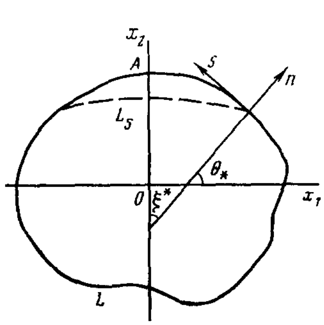

Fig.1

$$\mu\omega\left\{\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}r^m\varphi_{,\alpha n\ldots n}^{(m+1)} + \varepsilon_{\beta\alpha}x_\beta^{(0)} - \varepsilon_{\beta\alpha}n_\beta r\right\}\Big|_L = ks_\alpha \tag{2.6}$$

where $\varphi_{,\alpha n\ldots n}^{(m+1)}$ denotes the m-th derivative along the normal to L, of the first derivative of φ with respect to x_α.

Multiplying the left- and right-hand side of (2.6) by n_α and summing the result over the repeated index α, we obtain

$$\left\{\sum_{m=0}^{\infty} \frac{(-1)^m}{m!} r^m \varphi^{(m+1)}_{,n\ldots n} + \varepsilon_{\beta\alpha} n_\alpha x^{(0)}_\beta\right\}\Big|_L = 0 \tag{2.7}$$

Multiplying the equation (2.6) by s_α and repeating the above procedure, we arrive at the following expression:

$$\mu\omega\left\{\sum_{m=0}^{\infty} \frac{(-1)^m}{m!} r^m \varphi^{(m+1)}_{,sn\ldots n} + \varepsilon_{\beta\alpha} s_\alpha x^{(0)}_\beta - r\right\}\Big|_L = k \tag{2.8}$$

Thus we have obtained two conditions, (2.7) and (2.8), at the boundary, for solving the Laplace equation

$$\Delta\varphi = 0 \tag{2.9}$$

and determining the unknown function $r(\theta)$ defining the boundary L_S is the problem of torsion of an elasto-plastic bar.

Let the solution sought be dependent on some parameter δ. We shall seek the solution in the form of a power series in terms of this parameter

$$\varphi(\delta, \theta) = \sum_{i=0}^{\infty} \varphi_i \delta^i = \varphi_0 + \delta\bar{\varphi} \tag{2.10}$$

where φ_0 is the St. Venant stress function corresponding to the twist ω_0. We write the equations of the elasto-plastic boundary L_S in the form

$$x_\alpha(\delta, \theta) = \sum_{i=0}^{\infty} x^{(i)}_\alpha \delta^i = x^{(0)}_\alpha - \delta\bar{r} n_\alpha, \quad \bar{r} = \sum_{i=0}^{\infty} r_{i+1}\delta^i \tag{2.11}$$

Let further

$$\omega = \sum_{i=0}^{\infty} \omega_i \delta^i = \omega_0 + \delta\bar{\omega} \tag{2.12}$$

Substituting the expansions (2.10)—(2.12) into (2.7), (2.8) and (2.9) we obtain, respectively,

$$\left\{\sum_{m=1}^{\infty} \frac{(-1)^m}{m!} \delta^m \bar{r}^m \varphi^{(m+1)}_{0,n\ldots n} + \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \delta^{m+1} \bar{r}^m \varphi^{(m+1)}_{,n\ldots n}\right\}\Big|_L = 0 \tag{2.13}$$

$$\mu(\omega_0 + \delta\bar{\omega})\left\{\sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \delta^m \bar{r}^m (\varphi_0 + \delta\bar{\varphi})^{(m+1)}_{,sn\ldots n} + \varepsilon_{\beta\alpha} s_\alpha x^{(0)}_\beta - \delta\bar{r}\right\}\Big|_L = k \tag{2.14}$$

$$\Delta\varphi_i = 0 \quad i = 0, 1, 2, \ldots \tag{2.15}$$

Let us now assume that the quantity $\xi_* = \pi/2 - \theta_*$ (Fig.1) is small and of order δ. This implies that s_2 and n_1 are also of order δ. Consider the following expression:

$$F(\theta) = \{\mu\omega_0(\varphi_{0,s} + x_\alpha^{(0)} n_\alpha) - k\}\,|_L \tag{2.16}$$

Expanding the function $F(\theta)$ into a Taylor series along the arc of the contour L in the neighborhood of A, we have

$$F(\theta) = \sum_{m=0}^{\infty} \frac{1}{m!} F^{(m)}_{,S\ldots S}(\theta)\,|_{S=S_A} (S - S_A)^m \tag{2.17}$$

$$F(\theta)\,|_{S=S_A} = \{\mu\omega_0(\varphi_{0,s} + x_\alpha^{(0)} n_\alpha) - k\}\,|_{S=S_A} = 0$$

The tangential stress at the contour L assumes its maximum value at the point A, i.e.

$$F_{,s}(\theta)\,|_{S=S_A} = \mu\omega_0(\varphi_{0,s} + x_\alpha^{(0)} n_\alpha)_{,s}\,|_{S=S_A} = 0 \tag{2.18}$$

and we can therefore write (2.17) in the form

$$F(\theta) = \sum_{m=2}^{\infty} \frac{1}{m!} F^{(m)}_{,S\ldots S}\,|_{S=S_A} (S - S_A)^m \tag{2.19}$$

Let R be a finite parameter such that

$$S - S_A = R\xi = R(\pi/2 - \theta)$$

Then we can write (2.19) in the form

$$F(\theta) = \sum_{m=2}^{\infty} \frac{1}{m!} F^{(m)}_{,S\ldots S}\big|_{S=S_A} R^m (\pi/2 - \theta)^m$$

But $\xi = \pi/2 - \theta$ when $\theta \in [\theta_*, \pi/2]$ is a quantity of order δ, therefore the function $F(\theta)$ is of order δ^2. Equating in (2.13) the terms accompanying the first power of δ, we obtain

$$\{\varphi_{1,n} - \tau\varphi_{0,nn}\}\big|_L = 0 \tag{2.20}$$

Using (1.3)—(1.5) which hold for the elastic and plastic regions, we can write $\varphi_{0,\alpha\alpha} = 0$ and the condition (2.18) should hold at the point A:

$$\mu\omega_0(\varphi_{0,S} + x_\alpha^{(0)} n_\alpha),\ s\big|_{S=S_A} = \varphi_{0,SS}\big|_{S=S_A} = 0$$

Repeating the arguments used above we conclude, that $\varphi_{0,nn}$ on the arc $S_A S$ a quantity is of order δ, and hence $\varphi_{1,n} = 0$. Since r_1 is a function of the angle θ, we obtain $r_1 = \omega_1 = \varphi_1 = 0$, i.e. the expansion (2.10), (2.11) into series in small parameter of the solutions sought begin with the second power of δ, namely

$$\varphi(\delta, \theta) = \varphi_0(\theta) + \sum_{i=2}^{\infty} \varphi_i(\delta, \theta)\delta^i = \varphi_0 + \delta^2\bar{\varphi} \tag{2.21}$$

$$x_\alpha(\delta, \theta) = x_\alpha^{(0)}(\theta) - \sum_{i=2}^{\infty} r_i(\delta, \theta) n_\alpha \delta^i = x_\alpha^{(0)} - \delta^2 \bar{r} n_\alpha$$

We can assume without loss of generality that

$$\omega = \omega_0(1 + \delta^2) \tag{2.22}$$

and thus define the small parameter δ which remain undefined up to now. With (2.21) and (2.22) taken into account, the relations (2.13)—(2.15) become, respectively,

$$\left\{\sum_{m=1}^{\infty} \frac{(-1)^m}{m!} \delta^{2m} \bar{r}^m \varphi_{0,n\ldots n}^{(m+1)} + \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \delta^{2(m+1)} \bar{r}^m \bar{\varphi}_{,n\ldots n}^{(m+1)}\right\}\bigg|_L = 0 \tag{2.23}$$

$$\mu\omega_0(1 + \delta^2)\left\{\sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \delta^{2m} \bar{r}^m (\varphi_0 + \delta^2\bar{\varphi})_{,sn\ldots n}^{(m+1)} + \varepsilon_{\beta\alpha} s_\alpha x_\beta^{(0)} - \delta^2 \bar{r}\right\}\bigg|_L = k \tag{2.24}$$

$$\Delta\varphi_i = 0 \quad i = 0, 1, 2, \ldots \tag{2.25}$$

3. We consider, as an example, the problem of determining the boundary L_S for the case when a rectilinear bar of elliptical cross section is subjected to elasto-plastic torsion. The equation of the contour L in the $x_1 0 x_2$ plane will be

$$x_1^2/a_1^2 + x_2^2/a_2^2 = 1 \tag{3.1}$$

When $\omega = \omega_0$, a stress appears at two points of the contour L the components of which satisfy the equation (1.7). The coordinates of these points are $(0, a_2)$ and $(0, -a_2)$. The St. Venant function $\varphi_0(x_1, x_2)$ and the greatest tangential stress on L are given, respectively, by

$$\varphi_0(x_1, x_2) = -\frac{a_1^2 - a_2^2}{a_1^2 + a_2^2} x_1 x^2 \tag{3.2}$$

$$\tau_{1\,\max}\big|_L = -2\mu\omega_0 \frac{a_1^2 a_2}{a_1^2 + a_2^2} = -k \tag{3.3}$$

Equating in (2.23) the terms accompanying the second power of δ, we obtain

$$\varphi_{2,n}(\delta, \theta)\big|_L = 0 \tag{3.4}$$

The solution of the Laplace equation (2.25) with the boundary condition (3.4) will be

$$\varphi_2(\delta, \theta) = \text{const} \tag{3.5}$$

Equating in (2.24) the terms accompanying the second power of δ and taking into account (3.3) and (3.5), we obtain

$$\left\{\frac{2}{\delta^2}\left(\frac{a_1^2 a_2}{a_1^2+a_2^2}-\frac{a_1^2 x_2^{(0)}\sin\theta+a_2^2 x_1^{(0)}\cos\theta}{a_1^2+a_2^2}\right)-2\frac{a_1^2 x_2^{(0)}\sin\theta+a_2^2 x_1^{(0)}\cos\theta}{a_1^2+a_2^2}+r_2(\delta,\theta)\left(1-\frac{a_1^2-a_2^2}{a_1^2+a_2^2}(\cos^2\theta-\sin^2\theta)\right)\right\}\Bigg|_L=0 \tag{3.6}$$

$$x_1^{(0)}=\frac{a_1^2\cos\theta}{\sqrt{a_1^2\cos^2\theta+a_2^2\sin^2\theta}},\qquad x_2^{(0)}=\frac{a_2^2\sin\theta}{\sqrt{a_1^2\cos^2\theta+a_2^2\sin^2\theta}} \tag{3.7}$$

Here $x_1^{(0)}(\theta)$ and $x_2^{(0)}(\theta)$ are the contour coordinates of the cross section of bar L. Substituting (3.7) into (3.6) and taking into account the assumptions made before about the components of the unit vectors s_2 and n_1, we obtain

$$2\frac{a_1^2 a_2}{a_1^2+a_2^2}-\frac{a_1^2(a_1^2-a_2^2)}{a_2(a_1^2+a_2^2)}\frac{\cos^2\theta}{\delta^2}-r_2\frac{2a_1^2}{a_1^2+a_2^2}=0$$

and this yields

$$r_2(\delta,\theta)=a_2-\frac{a_1^2-a_2^2}{2a_2}\frac{\cos^2\theta}{\delta^2} \tag{3.8}$$

The elasto-plastic boundary in the present problem is symmetrical about the coordinate axes, therefore we shall study its behavior, in what follows, only in the first quadrant of $x_1 0 x_2$. It is clear that at some value of the angle $\theta=\theta_*^{(2)}$ the boundaries L and L_S will have a common point, i.e.

$$x_\alpha(\delta,\theta_*^{(2)})-x_\alpha^{(0)}(\theta_*^{(2)})=0$$

and this yields

$$\theta_*^{(2)}=\arccos\left(\delta\sqrt{\frac{2a_2^2}{a_1^2-a_2^2}}\right)=\frac{\pi}{2}-\delta\sqrt{\frac{2a_2^2}{a_1^2-a_2^2}}+O(\delta^2) \tag{3.9}$$

Equating in (2.23) the terms accompanying the third power of δ, we obtain

$$\left\{\varphi_{3,n}-r_2\frac{\varphi_{0,nn}}{\delta}\right\}\Bigg|_L=0$$

or, taking into account the assumptions made before,

$$\varphi_{3,n}|_L=\frac{(a_1^2-a_2^2)^2}{a_2(a_1^2+a_2^2)}\sin\theta\frac{\cos^3\theta}{\delta^3}-2\frac{a_2(a_1^2-a_2^2)}{a_1^2+a_2^2}\sin\theta\frac{\cos\theta}{\delta} \tag{3.10}$$

We know /5/ that the problem of torsion can be assumed solved if a function mapping a singly connected region D onto a circle is found. In the present case the relation mapping D onto the circle $|\zeta|<1$ is

$$z=x_1+ix_2=\omega(\zeta)=R\left(\zeta+\frac{m}{\zeta}\right),\qquad \zeta=\rho e^{i\alpha}$$

The boundary condition (3.10) can now be written as

$$\begin{aligned}&\varphi_{3,\rho}|_L=(3B/8-D/2)(\sigma+\bar\sigma)+B(\sigma^3+\bar\sigma^3)/8,\quad \alpha\in[\alpha_*,\ \pi-\alpha_*]\\&\varphi_{3,\rho}|_L=(D/2-3B/8)(\sigma+\bar\sigma)-B(\sigma^3+\bar\sigma^3)/8,\ \alpha\in[\pi+\alpha_*,\ 2\pi-\alpha_*]\\&B=\frac{1}{\delta^3}\frac{a_2^2(a_1^2-a_2^2)^2}{2a_1^2(a_1^2+a_2^2)},\quad D=\frac{1}{\delta}\frac{a_2^2(a_1^2-a_2^2)}{a_1^2+a_2^2},\quad \sigma=e^{i\alpha}\end{aligned} \tag{3.11}$$

and from /5/ we know that if

$$F_3=\varphi_3+i\psi_3 \tag{3.12}$$

where ψ_3 is a harmonic conjugate of φ_3, then

$$\zeta F_3'=\frac{1}{\pi i}\int\limits_{|\zeta|=1}\frac{\varphi_{3,\rho}|_L}{\sigma-\zeta}\,d\sigma \tag{3.13}$$

Substituting into (3.13) the boundary condition (3.11), we obtain

$$\begin{aligned}\zeta F_3'=&\frac{1}{2\pi i}\left\{C\left(\zeta+\frac{1}{\zeta}\right)+L^\circ\left(\zeta^3+\frac{1}{\zeta^3}\right)\ln\frac{(\sigma_*+\zeta)(\bar\sigma_*+\zeta)}{(\sigma_*-\zeta)(\bar\sigma_*-\zeta)}\right\}-\\&\frac{L^\circ}{2\pi i}\left\{2\left(\zeta^2+\frac{1}{\zeta^2}\right)(\sigma_*+\bar\sigma_*)-\frac{4}{3}(\sigma_*^3+\bar\sigma_*^3)\right\}\\&C=3B/8-D/2,\quad L^\circ=B/8\end{aligned} \tag{3.14}$$

From (3.12) we see that the following relations must hold at the boundary of the circle $|\zeta| = 1$

$$\frac{1}{2}\{\sigma F_3'(\delta,\sigma) + \overline{\sigma F_3'(\delta,\sigma)}\} = \varphi_{3,\rho}|_{\rho=1} \qquad (3.15)$$
$$\frac{i}{2}\{\sigma F_3'(\delta,\sigma) - \overline{\sigma F_3'(\delta,\sigma)}\} = \varphi_{3,\alpha}|_{\rho=1}$$

Using (3.14) we can write the second condition of (3.15) as

$$\varphi_{3,\alpha}|_{\rho=1} = -\frac{2B}{\pi}\left(\cos^2\alpha\cos\alpha_* - \frac{2}{3}\cos^3\alpha_*\right)$$

or, using the Cartesian coordinate system, as

$$\varphi_{3,S}|_L = -\frac{(a_1^2-a_2^2)^2}{\pi a_2(a_1^2+a_2^2)}\left(\frac{\cos^2\theta\cos\theta_*^{(2)}}{\delta^3} - \frac{2}{3}\frac{\cos^3\theta_*^{(2)}}{\delta^3}\right) \qquad (3.16)$$

Equating now in (2.24) the terms accompanying δ^3, we obtain

$$\{\varphi_{3,S} - r_3(1+\varphi_{0Sn})\}|_L = 0$$

or, taking account of the assumptions made above,

$$\varphi_{3,S} = \frac{2a_1^2}{a_1^2+a_2^2} r_3(\delta,\theta) \qquad (3.17)$$

Substituting (3.17) into (3.16) we find $r_3(\delta,\theta)$

$$r_3(\delta,\theta) = \frac{(a_1^2-a_2^2)^2}{2\pi a_1^2 a_2}\left(\frac{2}{3}\frac{\cos^3\theta_*^{(2)}}{\delta^3} - \frac{\cos^2\theta\cos\theta_*^{(2)}}{\delta^3}\right)$$

When $\theta = \theta_*^{(3)}$, the boundaries L and L_S will have a common point, i.e.

$$\delta^2 r_2(\delta,\theta_*^{(3)}) + \delta^3 r_3(\delta,\theta_*^{(3)}) = 0$$

and this yields

$$\theta_*^{(3)} = \arccos\left\{\delta\sqrt{\frac{2a_2^2}{a_1^2-a_2^2}}\left[1+\frac{2}{3}\delta\frac{a_2\sqrt{2(a_1^2-a_2^2)}}{\pi a_1^2}\right]^{1/2} \times \left[1+\delta\frac{a_2\sqrt{2(a_1^2-a_2^2)}}{\pi a_1^2}\right]^{-1/2}\right\} = \pi/2 - (\delta C_1 + \delta^2 C_2 + \delta^3 C_3) + O(\delta^4)$$

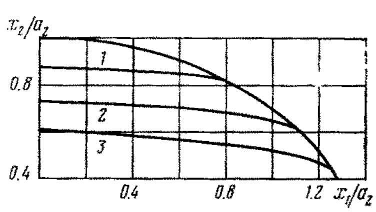

Fig.2

$$C_1 = \left(\frac{2a_2^2}{a_1^2-a_2^2}\right)^{1/2}, \quad C_2 = -\frac{a_2^2}{3\pi a_1^2}, \quad C_3 = \frac{\sqrt{2}a_2^3}{12\pi a_1^4}\frac{3(a_1^2-a_2^2)^2+4\pi^2 a_1^4}{(a_1^2-a_2^2)^{3/2}}$$

Fig.2 depicts the distribution of the boundary L_S for the following values of the critical angle of inclusion $\theta_*^{(3)}$ of the contour by the plastic region and the parameter δ:

1) $\theta_*^{(3)} = 1.032$ and $\delta = 0.35$;
2) $\theta_*^{(3)} = 0.875$ and $\delta = 0.5$;
3) $\theta_*^{(3)} = 0.74$ and $\delta = 0.6$.

Fig.3 shows the dependence of $[\tau_n]/k$ and $[\tau_S]/k$ on θ for the following values of $\theta_*^{(3)}$ and δ:

1) $\theta_*^{(3)} = 0.74$ and $\delta = 0.6$;
2) $\theta_*^{(3)} = 0.876$ and $\delta = 0.5$.

Here $\tau_n = \tau_\alpha n_\alpha$, $\tau_S = \tau_\alpha s_\alpha$, while n_α and s_α are the components of the unit vectors normal and tangent to the contour L. The values of τ_α were obtained using the relations (1.5) and (3.14).

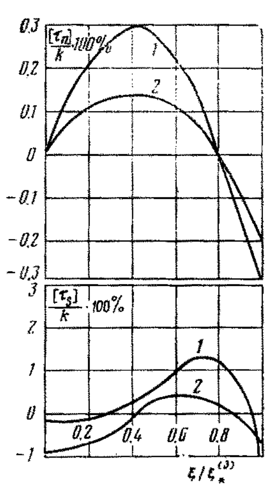

Fig.3

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